Canon No. 1. a 2 cancrizans
The Musical Offering, J. S. Bach

History
In 1747 J. S. Bach responded to Frederick the Great's invitation and paid a visit to Prussia's "Philosopher King." Bach's harpsichordist son, Carl, was employed by Frederick, and the obligation provided opportunity for the elder Bach to meet his new granddaughter.

Having arrived in Potsdam, the composer was summoned to the imperial palace where Frederick, an amateur flutist, interrupted a rehearsal in progress with, "Gentlemen, Old Bach has arrived." Johann Sebastian was then asked to test Frederick's new Silbermann fortepianos, and encouraged to improvise a three-voiced fugue upon a theme of the King's invention (the "Royal Theme"), which the composer did tout de suite. Amazed, Frederick then suggested a fugue in six voices, to which Bach demurred, this would require preparation.

Upon return to Leipzig, Bach developed the Royal Theme in a series of ten canons, two fugues (one in three voices and another in six), and a “church” sonata, sending them to his sovereign under the banner, A Musical Offering. (Note that one of the canons, and two of the trio sonata movements, are fugues. So there are actually five fugues in The Musical Offering.)

The above, with its "Royal Theme" in the top line, is the first of the Musical Offering's ten canons. It is a cancrizans, or "crab canon," a type of puzzle music that requires two musicians to play from opposite directions—one from the beginning forward, and the other from the end backward. Reaching the other pole, the players would reverse directions toward where each had begun, each then having played the music forward and in retrograde.

Structure
Gamesmanship aside, this canon represents one of the 18th century's most complex archetypes of double counterpoint. The following table, where each column represents a scheme of simultaneously sounding voices, reveals what happens in but the counterpoint of the first measure (m. 1) with the last (m. 18). This pattern is marvelously repeated in eight more measured pairs, gradually moving toward the center! The arrows represent forward (→) and backward (←) readings of the targeted measures.

<table>
<thead>
<tr>
<th>High Voice</th>
<th>m. 1 →</th>
<th>m. 18 →</th>
<th>m. 18 ←</th>
<th>m. 1 ←</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Voice</td>
<td>m. 18 ←</td>
<td>m. 1 ←</td>
<td>m. 1 →</td>
<td>m. 18 →</td>
</tr>
</tbody>
</table>

Three-Dimensional Model
To make your canonic Möbius strip: (1) Cut along the solid lines of the canon's rectangle, (2) Fold along the dotted line so that the music is visible on both sides, (3) Wrap the strip, giving it a half twist, then tape the ends together so that the backward and forward clefs are next to each other. To "perform" your model, one musician would start at each clef, read around the strip in the prescribed direction to the other clef, and then reverse directions.

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More on Bach’s Musical Offering at http://www2.nau.edu/tas3/musoffcanons.html
See the animation of this canon at http://strangepaths.com/canon-1-a-2/2009/01/18/en/